BULGARIAN ACADEMY OF SCIENCES
SPACE RESEARCH IN BULGARIA, 1
Sofia · 1978

Discriminant Analysis of Natural Formations Reflective Characteristics by a Minimal Number of Wavelengths

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It has been established that the different natural formations reflect the energy of the visible and infrared ranges in different ways. As the range is comparatively broad the spectral reflective characteristics are measured in a considerable number of wavelengths. That makes the measuring process and the subsequent analysis difficult, and the information obtained contains a big amount of redundancy.

The problem to be solved here is the following: what is the minimum number of wavelengths λ_i and exactly in which wavelengths the reflection index must be measured so that the identification of the reflective characteristics $r_j(\lambda_i)$ of a set M, given in advance, from J classes of objects O_j, $j=1, \ldots, J$, can be ensured. We assume that the functions $r_j(\lambda_i)$, $i=1, \ldots, n$ are given with their confidence intervals $\pm \Delta r_j(\lambda_i)$ in the visible range of electromagnetic waves $\Delta\lambda_v=\lambda_n-\lambda_1$. It is assumed that $r_j(\lambda_i)$ are stationary random functions. The problem is solved in two ways: (1) the reflective spectral characteristics are used directly for the purposes of identification; and (2) a transformation of $r_j(\lambda_i)$ is carried out in advance by means of suitable transforming functions, after which the identification of the transformed functions is performed.

1. Identification by Means of $r_j(\lambda_i)$

The dividing surface for the identification of $r_j(\lambda_i)$ is chosen in accordance with the Bayes criterion for a minimum average risk (one-dimensional case):

$$\lambda_0=\frac{w(\lambda_i|a_k)}{w(\lambda_i|a_j)}=\frac{r_{kj}\,p(a_j)}{r_{jk}\,p(a_k)}, \tag{1}$$

where r_{jk} and r_{kj} are weighting coefficients of the j and k classes of objects, $p(a_j)$ and $p(a_k)$ are a priori probabilities of appearance in these classes. In our case all objects will be considered as equally likely and of equal weight and then $\lambda_0 = 1$, i. e. the dividing surface of the criterion is reduced to the intersection point of the $w(\lambda_i/a_k)$ and $w(\lambda_i/a_j)$ distributions of j and k classes. If this point is out of the confidence limits $\pm\Delta r_j(\lambda_i)$ and $\pm\Delta r_k(\lambda_i)$ of the classes compared, these classes are identified in the wavelength λ_i. The algorithm is easily realized in the following way:

The matrix A of the relations "the ith and the kth classes are recognizable in the $r_j \pm \Delta r_j$ and $r_k \pm \Delta r_k$ intervals" (these relations are denoted here with 1). The opposite ratios of nonrecognizability (when $r_j \pm \Delta r_j$ and $r_k \pm \Delta r_k$ have a non-empty cross section) are designated by 0. The matrix A is formed for all two-element combinations C_j^2 of the O_j classes and for all wavelengths λ_i.

$A =$

λ_i \ C_j^2	O_{11}	O_{12}	...	O_{1n}	O_{21}	...	O_{2n}	...
λ_1	0	1	...	0	0	...	0	...
...	...	...	...	...	0	...	...	...
λ_n	1	1	...	0	1	...	1	...

A subset of M is identified in a given combination of wavelengths λ_i, if in each column of A containing the wavelengths there is at least one number 1. In this way the rule for the addition of 0 and 1 is determined: $0+0=0$; $0+1=1+0=1+1=1$. The identification of $r_j(\lambda_i)$ by means of the combination C_n^m of the wavelengths is possible only if all columns of the submatrix formed by C_n^m have a sum equal to 1.

The complete solution of the problem is obtained by studying successively the combinations C_n^m, $m = 1, \ldots, n$ until the first solutions belonging to a given class of wavelength combinations C_n^m are obtained.

2. Identification by Means of Transformation in Advance

The transformation of the original function $r_j(\lambda_i)$ is reasonable if the new function offers better possibilities of identifying $r_j(\lambda_i)$ by means of a smaller number of wavelengths. That is why it is necessary for the transformed function $z = f(r_j)$ to check whether the Bayes criterion shows better results (reducing Type I and Type II errors). For this purpose the law of the distribution of z must be determined:

$$p(z) = p[\Psi(z)] \,.\, \Psi'(z), \quad r_j = \Psi(z); \quad \Psi'(z) = \frac{d\Psi}{dz}. \tag{2}$$

After that for the z_j and z_k distributions of the two classes compared the summary error of Type I and Type II is to be found:

$$p = \int_{-\infty}^{z_{cr}} p_k(z)dz + \int_{z_{cr}}^{\infty} p_j(z)dz, \tag{3}$$

where z_{cr} — internal intersection point of $p(z_j)$ and $p(z_k)$.

In general, the integral in (3) is not solvable. When the set is given it can be solved by numerical methods. In order to obtain some analytical results we examined the particular case determined by the following limiting conditions:

I. The distribution $p(r_j)$ is normal;

II. The conditions $\sigma_j \ll \mu_j$ and, in particular, $\sigma_j = q\mu_j$, $q \ll 1$, are satisfied.

III. The functions z are integral transformations which, according to r_j and r_k sampling, are replaced by their sums.

In this study the following transforming functions are examined:

a) Module-structure function:

(4)
$$C(\tau) = \int_0^T |f(x) - f(x+\tau)|\, dx$$

used in paper [1]. There it is proved that the function is symmetric with a symmetry axis at $x = \frac{T}{2}$. It is shown also that $C(\tau)$ and $f(x)$ are in a homomorphous relationship, i. e. different functions determined in the following way: $f(x)$, $if(jx+C)+D$, $i=\pm 1$, $j=\pm 1$, $C=\text{const.}$, $D=\text{const.}$ may correspond to the same function $C(\tau)$. This homomorphism is not a strong limitation, because in practice it can be easily transformed into an isomorphism by means of control characteristics of the type: $C_{\text{cntr}}(\tau) = \int_0^{x_{ct}} |f(x) - f(x+\tau)| dx$ and by the $f(x)$ value for $x = x_{ct}$. For the example examined in this work it is not necessary to use such control characteristics.

b) Kolmogorov's structural function:

(5)
$$C_K(\tau) = \int_0^T [f(x) - f(x+\tau)]^2 dx;$$

c) Autocorrelation function;

(6)
$$K(\tau) = \int_0^T [f(x) - m][f(x+\tau) - m] dx.$$

In our case $f(x) = r(\lambda_i)$ and the integrals are substituted by the following sums:

(4a)
$$C_j(\tau) = \sum_{(i)} [r_j(\lambda_i) - r_j(\lambda_j + \tau)],$$

(5a)
$$C_{kj}(\tau) = \sum_{(i)} [r_j(\lambda_i) - r_j(\lambda_i + \tau)]^2$$

(6a)
$$K_j(\tau) = \sum_{(i)} [r_j(\lambda_i) - \bar{r}_j][r_j(\lambda_i + \tau) - \bar{r}_j].$$

It follows from the limiting conditions I, II, III that the sum $C_j(\tau)$ has also a normal distribution with an arithmetic mean C_j and dispersion σ_j^2 respectively equal to:

$$(7) \qquad \overline{C_j(\tau)} \approx \sum_{(i)} [\overline{r_j(\lambda_i)} - \overline{r_j(\lambda_i+\tau)}]; \quad \sigma_j^2 \approx \sum q^2 \{[\overline{r_j(\lambda_i)}]^2 + [\overline{r_j(\lambda_i+\tau)}]^2\}.$$

For the sums C_{kj} and K_j, according to equations (5a) and (6a) and conditions I and II, after neglecting the small terms of higher order, we obtain:

$$(8) \qquad \overline{C_{kj}(\tau)} \approx \sum_{(i)} [\overline{r_j(\lambda_i)} - \overline{r_j(\lambda_i+\tau)}]^2$$

$$\sigma_{kj}^2 \approx \sum_{(i)} 4q^2 [\overline{r_j(\lambda_i)} - \overline{r_j(\lambda_i+\tau)}] \{[\overline{r_j(\lambda_i)}]^2 + [\overline{r_j(\lambda_i+\tau)}]^2\}$$

and

$$(9) \qquad K_j \approx \sum_{(i)} [\overline{r_j(\lambda_i)} - \overline{r_j}][\overline{r_j(\lambda_i+\tau)} - \overline{r_j}]$$

$$\sigma_{kj}^2 \approx \sum_{(i)} \{[\overline{r_j(\lambda_i)} - \overline{r_j}]^2 \overline{r_j(\lambda_i)}^2 + [\overline{r_j(\lambda_i+\tau)} - \overline{r_j}]^2 \overline{r_j(\lambda_i+\tau)}^2\} q^2.$$

Thus the distribution of C, C_k, K from equations (4a), (5a) and (6a) proves to be normal, provided the limitations I and II are given. Then their intersection point for the j and k classes is found by means of the equation:

$$\frac{1}{\sqrt{2\pi}\,\sigma_j} e^{-\frac{(r-\mu_j)^2}{2\sigma_j^2}} = \frac{1}{\sqrt{2\pi}\,\sigma_k} e^{-\frac{(r-\mu_k)^2}{2\sigma_k^2}}$$

After calculating its logarithm the equation takes the following form:

$$\frac{(r-\mu_j)^2}{2\sigma_j^2} = \ln \frac{\sigma_j}{\sigma_k} - \frac{(r-\mu_k)^2}{2\sigma_k^2},$$

thus

$$(10) \qquad r_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

where $a = \frac{1}{\sigma_k^2} - \frac{1}{\sigma_j^2}$,

$$b = 2\left(\frac{\mu_j}{\sigma_j^2} - \frac{\mu_k}{\sigma_k^2}\right),$$

$$c = \frac{\mu_k^2}{\sigma_k^2} - \frac{\mu_j^2}{\sigma_j^2} - 2\ln\frac{\sigma_j}{\sigma_k}.$$

The efficiency of each of the transformations C, C_k, K is measured by the value of the integral in (3). In this case it may take the following form:

(3a) $$p = \int_{-\infty}^{\eta_{j\,1,2}} e^{-\frac{\eta_j^2}{2}}\, d\eta_j + \int_{\eta_{k1,2}}^{\infty} e^{-\frac{\eta_k^2}{2}}\, d\eta_k ,$$

where $\eta_{j,k} = \frac{r - \mu_{j,k}}{\sigma_{j,k}}$ is the coordinate of the normalized normal distribution. The value of p will be smaller when the limit

(11) $$\eta_{1,2_{j,k}} = \frac{r_{1,2} - \mu_{j,k}}{\sigma_{j,k}}$$

is greater in absolute value. In spite of the limiting conditions I, II and III, the analytical examination of the criterion (11) is still difficult, as the expressions for $\mu_{j,k}$ and $\sigma_{j,k}$ from (7), (8) and (9) take part in (10) in a relatively complicated manner. That is why two particular problems are treated in our further work.

IV. The functions $\overline{r_j(\lambda_i)}$ and $\overline{r_k(\lambda_i)}$ are connected by the determinated functional relation:

(12) $$\overline{r_k(\lambda_i)} = (1+\theta)\overline{r_j(\lambda_i)},$$

where $\theta = \text{const}$, $|\theta| \ll 1$.

V. The functions $\overline{r_k(\lambda_i)}$ and $\overline{r_j(\lambda_i)}$ are of such a type that their differences are of a random character.

(13) $$\overline{r_k(\lambda_i)} = \overline{r_j(\lambda_i)} + \Delta r_{j,k}(\lambda_i),$$

where $\Delta r_{j,k}(\lambda_i)$ — random function;

$\overline{\Delta r_{j,k}(\lambda_j)} = 0.$

We shall first examine case IV.

It follows from equations (7), (8) and (9) that for μ, σ defined by (12) and for the three transforming functions C, C_k, and K the following expressions are valid:

$$\mu_k = (1+\theta)\,\mu_j, \quad \sigma_k^2 = (1+\theta)^2\,\sigma_j^2.$$

Then the equation (10) takes the form

$$r = -\frac{b}{a} = \mu_j \frac{2(1+\theta)}{2+\theta}$$

assuming $\ln \sigma_k/\sigma_j \approx 0$, and (11), respectively:

(11a) $$\eta_{j,k} = \frac{\mu_j}{\sigma_j} \cdot \frac{\theta}{2+\theta} = \frac{\theta}{2+\theta} \cdot \frac{1}{V_j}; \quad V_j = \frac{\sigma_j}{\mu_j}.$$

Therefore in this case the efficiency of the criterion η is inversely proportional to the coefficient of variance V and the comparative analysis is to be carried out by means of V_C, V_{C_K}, V_K. For this purpose the following ratios should be formed:

$$\omega_{C,C_K}=\frac{V_{C_K}}{V_C}=\frac{\sigma_{C_K}\cdot\bar{C}}{\bar{C}_K\cdot\sigma_C},\quad \omega_{C,K}=\frac{V_K}{V_C}=\frac{\sigma_K\cdot\bar{C}}{\bar{K}\cdot\sigma_C}.$$

With a view to equations (7), (8), (9) for ω_{C,C_K}:

$$(14)\qquad \omega_{C,C_K}=\sqrt{\frac{\left(\sum_{(i)}|X_i-Y_i|\right)^2\cdot 4\cdot\sum_{(i)}(X_i-Y_i)^2(X_i^2+Y_i^2)}{\left[\sum_{(i)}(X_i^2+Y_i^2)\right]\left[\sum_{(i)}(X_i-Y_i)^2\right]^2}}$$

is obtained where, for the sake of simplicity, the substitutions $X_i=r_j(\lambda_i)$, $Y_i=r_j(\lambda_i+\tau)$ are used. The sum $\Sigma|X_i-Y_i|$ can also be presented as follows:

$$\sum_{(i)}|X_i-Y_i|=\sum_{(M_1)}(X_i-Y_i)+\sum_{(M_2)}(Y_i-X_i),$$

where M_1 is the set of i values for which $X_i>Y_i$, and M_2 is the set of i values for which $X_i<Y_i$. This way of presenting allows $\Sigma|X_i-Y_i|$ to be differentiated with respect to X_i and Y_i respectively.

In such a case the extremums of ω_{C,C_K} are determined by the condition:

$$(15)\qquad \frac{\partial\omega_{C,C_K}}{\partial x_i}=0,\qquad (15a)\quad \frac{\partial\omega_{C,C_K}}{\partial y_i}=0,\ i=1,\ldots,n.$$

The system (15) is equivalent to the following system:

$$(16)\qquad \begin{aligned}&\{\pm 2\Sigma|X_i-Y_i|\cdot 4\cdot\Sigma[X_i-Y_i]^2(X_i^2+Y_i^2)+[\Sigma|X_i-Y_i|]^2[2(X_i-Y_i)(X_i^2+Y_i^2)\\&+(X_i-Y_i)^2 2X_i]\}\{\Sigma(X_i^2+Y_i^2)[\Sigma(X_i-Y_i)^2]^2\}-[\Sigma|X_i-Y_i|]^2 4\\&\times\Sigma[(X_i-Y_i)^2(X_i^2+Y_i^2)]\cdot\{2X_i[\Sigma(X_i-Y_i)^2]^2\\&+[\Sigma(X_i^2+Y_i^2)][2\Sigma(X_i-Y_i)^2][2(X_i-Y_i)]\}=0.\end{aligned}$$

In (16) all sums are identical for the equation system where $i=1,\ldots,n$. Therefore (16) is a system of equations of the third power with respect to X_i. This system is to be satisfied, i. e. the n equations of the third power must be cancelled out by their roots. As in this case all $X_i>0$, the equations of the third power have only one real positive root, i. e. $X_i=\text{const}=X$.

Similarly, the condition $Y_i=\text{const}=Y$ can be obtained which satisfies the system (15a).

Therefore ω_{C,C_K} has an extremum at $X_i=X$, $Y_i=Y$.

The value of the extremum is

$$(17)\qquad (\omega_{C,C_K})\,\text{ext}=2.$$

By way of example a check with the following values of X_i and Y_i: $x_1=a$, $x_2=2a$, $x_3=3a$, $y_1=2a$, $y_2=3a$, $y_3=a$ shows that the extremum is a maximum.

The following expression is obtained for the coefficient $\omega_{C,K}$

(18) $$\omega_{C,K}=\sqrt{\frac{\left(\sum_{(i)}|X_i-Y_i|\right)^2\sum_{(i)}[X_i^2(X_i-\bar{r}_j)^2+Y_i^2(Y_i-\bar{r}_j)^2]}{\sum_{(i)}(X_i^2+Y_i^2)\left[\sum_{(i)}(X_i-\bar{r}_j)(Y_i-\bar{r}_j)\right]^2}}$$

In a way analogous to the one used for ω_{C,C_K} it is shown that $\omega_{C,K}$ has an minimum at $X_i=X$, $Y_i=Y$. As $\sum_{(i)}X_i=\sum_{(i)}Y_i$, it follows that at the extremum $X=Y$, but then $\lim_{X_i\to Y_i}\bar{r}=\frac{x+y}{2}$ and, consequently $|x-r|=|y-r|=\frac{|-y+x|}{2}$. The value of the minimum in this case is

(19) $$(\omega_{C,K})\,\text{ext}=2.$$

Finally, let us determine the ratio $\omega_{C,r}$, measuring the efficiency of C with respect to r_j. According to I and II this ratio is

(20) $$\omega_{C,r}=\sqrt{\frac{\left(\sum_{(i)}|X_i-Y_i|\right)^2}{\sum_{(i)}(X_i^2+Y_i^2)}}.$$

The magnitude of $\omega_{C,r}$ depends on the concrete structure of $r_j(\lambda_i)$. For instance, for the straight line $y=x$ the module-structure characteristic is given by the expression $C(\tau)=2\tau(T-\tau)$, while for the sine-shaped curve $y=\sin x$ this expression is $C(\tau)=8\sin\frac{\tau}{2}$. It is clear that for the straight line the value of the function $C(\tau)$ becomes greater than half the area between the straight line and the abscissa when $\tau>\frac{T}{2}\left(1-\frac{1}{\sqrt{2}}\right)$ and, for the sine-shaped curve, when $\sin\frac{\tau}{2}>\frac{1}{2}$. Our experience shows that for curves of the spectral reflective characteristic type a considerable range of τ exists where the condition $\Sigma|r_j(\lambda_i)-r_j(\lambda_i+\tau)|>n\frac{\bar{r}_j}{2}$ is fulfilled. In this case, as $\sum_{(i)}X_i^2=\sum_{(i)}Y_i^2<nX_{\max}^2$ and $\bar{r}_j>\frac{X_{\max}}{2}$ for $\omega_{C,r}$ there exists a range for which

(21) $$\omega_{C,r}>\sqrt{\frac{n}{8}}.$$

So far we have examined the case defined by condition IV. Let us now assess the case defined by condition V. For this purpose it is sufficient to examine the difference $|C_{(K)}-C_{(j)}|$, $|C_{K(k)}-C_{K(j)}|$ and $|K_K-K_j|$. As the reflective characteristics r_k and r_j differ only in the random function which has arithmetic mean equal to zero for a sufficiently large set of values of i, the following is obtained for the above differences:

$$C_k - C_j = \sum_{(i)} |X_{K_i} - Y_{K_i}| - \sum_{(i)} |X_{j_i} - Y_{j_i}| \tag{22}$$

$$= \sum_{(M_1)} [\Delta X_{j_k}(i) - \Delta Y_{j_k}(i)] + \sum_{(M_2)} [\Delta Y_{j_k}(i) - \Delta X_{j_k}(i)] = 0$$

in accordance with condition II.
Similarly we obtain:

$$|C_{k(k)} - C_{k(j)}| = \sum_{(i)} [\Delta X_{jk}(i) - \Delta Y_{j_k}(i)]^2 \neq 0. \tag{23}$$

$$|K_k - K_j| = \sum_{(i)} [\Delta X_{j_k}(i)] [\Delta Y_{j_k}(i)] \neq 0. \tag{24}$$

Equations (22), (23) and (24) show that for functions defined by condition V the module-structure characteristics have 0 efficiency, as opposed to C_k and K. This result can be considered as a weighting property of C which C_k and K do not possess to the same degree.

Discussion

1. When the reflective characteristics are used for their identification directly the algorithm exposed in item A furnishes the answer as to what the minimum number is of wavelengths $\lambda_{i\,(\min)}$ by means of which the appurtenance of new objects to a given set M of reference classes can be recognized. If the number of these classes is not too large, the necessary number of wavelengths $\lambda_{i\,(\min)}$ in which r is to be measured is comparatively small. This simplifies the measuring process and the analysis of reflective characteristics.

2. In the case where M contains many reference classes (for instance, several hundred) it is probable that the number of $\lambda_{i\,(\min)}$ shall be commensurable with the total number n of the sampled values of r. Here it is advisable to use some of the transforming functions C, C_k, K examined above.

The equations (17), (19) and (21) contain the basic results of the three transforming functions obtained so far. They show that there is a possibility for the module-structure characteristics, defined by equation (4), to have a better efficiency than the original functions and the transformations defined by equations (5) and (6). This efficiency results in a decrease of probability for type I and type II errors using the Bayes criterion for a minimum risk when the identification of r_j is carried out. This reduction of errors leads to possibilities for the decrease also of the minimum number of $C(\tau)$ values, by means of which the identification of the set M of reference classes with given reflective characteristics is realized. A better efficiency of c is therefore to be looked for in the range of the greater values of τ where the coefficient of variance V_c decreases considerably.

Table 1

λ_i \ c. №	5	6	9	10	11	14	21	26	53	67	68	69	74	75	85	90	94
400	0.059	0.072	0.084	0.020	0.202	0.050	0.023	0.105	0.018	0.152	0.149	0.054	0.044	0.030	0.030	0.017	0.015
410	0.060	0.073	0.086	0.020	0.203	0.050	0.024	0.100	0.019	0.153	0.149	0.050	0.046	0.033	0.038	0.018	0.015
420	0.061	0.074	0.091	0.020	0.204	0.050	0.025	0.090	0.020	0.154	0.149	0.049	0.049	0.039	0.042	0.019	0.018
430	0.062	0.074	0.097	0.021	0.205	0.050	0.027	0.096	0.024	0.159	0.150	0.056	0.051	0.040	0.048	0.020	0.020
440	0.061	0.073	0.101	0.022	0.209	0.050	0.028	0.094	0.026	0.162	0.157	0.060	0.055	0.041	0.050	0.021	0.022
450	0.065	0.072	0.109	0.024	0.210	0.050	0.026	0.092	0.024	0.171	0.163	0.061	0.059	0.042	0.051	0.025	0.025
460	0.073	0.071	0.115	0.025	0.211	0.050	0.027	0.090	0.027	0.182	0.176	0.060	0.063	0.043	0.052	0.030	0.027
470	0.076	0.070	0.125	0.026	0.211	0.050	0.026	0.090	0.029	0.200	0.190	0.059	0.068	0.044	0.053	0.036	0.029
480	0.075	0.070	0.131	0.027	0.212	0.050	0.026	0.090	0.028	0.215	0.200	0.06[illegible]	0.071	0.045	0.052	0.039	0.027
490	0.087	0.071	0.144	0.027	0.214	0.050	0.026	0.091	0.027	0.222	0.209	0.066	0.075	0.045	0.052	0.040	0.026
500	0.095	0.073	0.161	0.029	0.215	0.050	0.022	0.095	0.027	0.277	0.218	0.070	0.081	0.049	0.055	0.041	0.029
510	0.106	0.077	0.185	0.034	0.215	0.051	0.031	0.100	0.027	0.230	0.222	0.070	0.089	0.059	0.062	0.044	0.035
520	0.120	0.080	0.213	0.040	0.215	0.053	0.036	0.105	0.029	0.231	0.232	0.096	0.096	0.075	0.075	0.050	0.050
530	0.149	0.082	0.233	0.053	0.214	0.056	0.041	0.111	0.033	0.232	0.226	0.110	0.103	0.087	0.093	0.060	0.072
540	0.169	0.087	0.245	0.066	0.213	0.059	0.051	0.114	0.041	0.234	0.227	0.119	0.109	0.094	0.107	0.071	0.093
550	0.176	0.090	0.249	0.070	0.213	0.060	0.049	0.114	0.045	0.235	0.228	0.120	0.112	0.098	0.112	0.095	0.104
560	0.178	0.091	0.247	0.067	0.213	0.061	0.048	0.112	0.044	0.239	0.230	0.112	0.116	0.096	0.113	0.129	0.097
570	0.173	0.095	0.241	0.053	0.216	0.063	0.050	0.108	0.042	0.243	0.232	0.111	0.118	0.088	0.111	0.154	0.084
580	0.168	0.098	0.231	0.048	0.220	0.064	0.044	0.103	0.040	0.246	0.235	0.112	0.117	0.081	0.110	0.151	0.073
590	0.168	0.100	0.229	0.050	0.224	0.065	0.043	0.100	0.039	0.248	0.238	0.105	0.115	0.081	0.109	0.143	0.067
600	0.160	0.101	0.224	0.050	0.231	0.066	0.044	0.097	0.040	0.248	0.239	0.088	0.113	0.085	0.105	0.132	0.061
610	0.152	0.101	0.220	0.049	0.239	0.067	0.042	0.094	0.050	0.246	0.239	0.078	0.118	0.085	0.101	0.128	0.060
620	0.153	0.102	0.211	0.053	0.247	0.068	0.040	0.090	0.054	0.243	0.237	0.078	0.125	0.079	0.096	0.130	0.059
630	0.151	0.102	0.205	0.051	0.254	0.067	0.043	0.087	0.052	0.239	0.231	0.078	0.132	0.073	0.091	0.131	0.052
640	0.143	0.101	0.202	0.042	0.261	0.071	0.043	0.083	0.050	0.233	0.228	0.074	0.140	0.069	0.087	0.129	0.047
650	0.141	0.100	0.208	0.036	0.269	0.080	0.035	0.080	0.048	0.229	0.222	0.082	0.142	0.075	0.085	0.121	0.040
660	0.150	0.100	0.211	0.040	0.274	0.089	0.025	0.075	0.050	0.224	0.218	0.095	0.148	0.082	0.087	0.123	0.045
670	0.161	0.100	0.218	0.055	0.280	0.097	0.022	0.072	0.053	0.228	0.224	0.101	0.151	0.105	0.089	0.138	0.045
680	0.178	0.100	0.225	0.068	0.284	0.102	0.028	0.085	0.060	0.242	0.230	0.112	0.157	0.145	0.095	0.161	0.045
690	0.210	0.100	0.241	0.088	0.288	0.106	0.030	0.122	0.070	0.251	0.245	0.132	0.165	0.182	0.105	0.201	0.042
700	0.238	0.100	0.252	0.120	0.295	0.107	0.042	0.180	0.110	0.268	0.252	0.151	0.172	0.212	0.122	0.248	0.061
710	0.264	0.100	0.271	0.141	0.298	0.106	0.050	0.245	0.122	0.298	0.261	0.180	0.185	0.148	0.141	0.308	0.072
720	0.291	0.100	0.301	0.164	0.299	0.106	0.062	0.278	0.150	0.329	0.272	0.259	0.210	0.291	0.158	0.342	0.100
730	0.332	0.100	0.310	0.187	0.301	0.111	0.070	0.328	0.161	0.343	0.303	0.342	0.232	0.301	0.184	0.395	0.125
740	0.374	0.100	0.321	0.218	0.301	0.122	0.075	0.369	0.180	0.350	0.308	0.467	0.250	0.306	0.218	0.405	0.172
750	0.418	0.100	0.351	0.270	0.300	0.145	0.079	0.397	0.200	0.350	0.306	0.542	0.252	0.305	0.269	0.470	0.205
760	0.458	0.101	0.360	0.320	0.300	0.190	0.081	0.418	0.202	0.348	0.303	0.571	0.272	0.303	0.299	0.481	0.249
770	0.512	0.102	0.370	0.342	0.299	0.220	0.082	0.437	0.201	0.339	0.300	0.591	0.281	0.309	0.322	0.491	0.292
780	0.517	0.103	0.398	0.360	0.296	0.240	0.083	0.452	0.193	0.327	0.293	0.605	0.289	0.311	0.342	0.501	0.322
790	0.526	0.104	0.405	0.375	0.293	0.255	0.084	0.470	0.189	0.311	0.283	0.615	0.293	0.313	0.359	0.520	0.342

Here it is appropriate to deal with the following cases:

a) When τ is small, a correlation between the neighbouring values of r is possible to exist, i. e. the difference $r_j(\lambda_i)-r_j(\lambda_i+\tau)$ will not be a composition of independent random quantities. Then the equations (7), (8) and (9) will not be valid. However, a similar correlation could hardly be expected for $\tau>(5\div8)\Delta\lambda=50-80$ nm because such a τ corresponds to a transition into a zone of a new hue. Because of that most values of C, C_k, K and their dispersions remain as defined in (7), (8) and (9).

b) As the efficiency of $C(\tau)$ is expected to be considerable when the τ values are higher, the following question is to be answered: When the set M is large, will there be a sufficient number of high values of C for the identification of the classes of M? The affirmative answer to this question

95	96	97	98	99	100	101	102	105	106	107	108	109	111	112	113	143
0.022	0.020	0.022	0.022	0.022	0.022	0.020	0.023	0.030	0.021	0.028	0.037	0.040	0.059	0.050	0.027	0.019
0.022	0.020	0.022	0.022	0.022	0.022	0.020	0.023	0.031	0.021	0.028	0.037	0.042	0.061	0.050	0.032	0.024
0.023	0.020	0.023	0.023	0.023	0.023	0.021	0.026	0.037	0.022	0.029	0.037	0.045	0.069	0.050	0.035	0.027
0.024	0.020	0.024	0.024	0.024	0.028	0.023	0.028	0.045	0.023	0.030	0.038	0.049	0.077	0.051	0.035	0.029
0.026	0.022	0.026	0.028	0.028	0.032	0.026	0.031	0.043	0.024	0.031	0.039	0.050	0.100	0.053	0.035	0.029
0.027	0.023	0.027	0.030	0.030	0.035	0.029	0.033	0.047	0.025	0.032	0.040	0.053	0.150	0.057	0.037	0.030
0.029	0.025	0.029	0.032	0.032	0.039	0.030	0.035	0.053	0.026	0.034	0.041	0.056	0.180	0.060	0.038	0.028
0.028	0.026	0.028	0.034	0.034	0.040	0.030	0.035	0.058	0.029	0.036	0.042	0.058	0.192	0.061	0.041	0.033
0.032	0.028	0.032	0.038	0.038	0.039	0.031	0.035	0.056	0.030	0.038	0.043	0.060	0.181	0.062	0.040	0.028
0.035	0.030	0.035	0.040	0.040	0.040	0.033	0.038	0.058	0.031	0.039	0.047	0.063	0.163	0.068	0.039	0.030
0.040	0.041	0.041	0.045	0.045	0.049	0.038	0.042	0.068	0.035	0.045	0.052	0.070	0.149	0.072	0.041	0.035
0.049	0.059	0.052	0.060	0.062	0.070	0.050	0.056	0.086	0.041	0.053	0.064	0.088	0.127	0.074	0.054	0.044
0.066	0.079	0.074	0.082	0.092	0.105	0.078	0.090	0.124	0.055	0.070	0.080	0.120	0.125	0.115	0.089	0.050
0.086	0.098	0.101	0.115	0.123	0.149	0.112	0.119	0.172	0.076	0.089	0.101	0.159	0.116	0.167	0.116	0.055
0.102	0.111	0.119	0.128	0.143	0.182	0.129	0.134	0.202	0.090	0.108	0.121	0.180	0.104	0.192	0.126	0.058
0.115	0.103	0.121	0.129	0.151	0.198	0.133	0.145	0.225	0.095	0.118	0.134	0.184	0.096	0.210	0.131	0.061
0.110	0.092	0.117	0.123	0.147	0.182	0.131	0.142	0.224	0.090	0.112	0.131	0.173	0.087	0.199	0.121	0.062
0.096	0.080	0.105	0.111	0.127	0.161	0.120	0.131	0.221	0.081	0.103	0.122	0.159	0.072	0.173	0.114	0.061
0.087	0.072	0.092	0.100	0.110	0.141	0.107	0.114	0.204	0.073	0.096	0.111	0.147	0.076	0.155	0.093	0.059
0.078	0.065	0.083	0.090	0.095	0.134	0.098	0.103	0.185	0.069	0.090	0.103	0.134	0.085	0.152	0.084	0.057
0.070	0.061	0.077	0.082	0.088	0.133	0.094	0.100	0.186	0.063	0.082	0.097	0.126	0.145	0.161	0.073	0.055
0.065	0.057	0.071	0.077	0.081	0.110	0.091	0.097	0.176	0.060	0.079	0.090	0.119	0.238	0.146	0.079	0.053
0.061	0.050	0.069	0.073	0.079	0.095	0.084	0.094	0.161	0.059	0.073	0.087	0.112	0.318	0.131	0.077	0.051
0.056	0.045	0.065	0.070	0.057	0.100	0.079	0.089	0.157	0.057	0.070	0.082	0.107	0.385	0.117	0.068	0.052
0.054	0.042	0.064	0.070	0.077	0.092	0.073	0.082	0.152	0.055	0.068	0.080	0.101	0.510	0.105	0.070	0.053
0.057	0.045	0.065	0.070	0.078	0.096	0.071	0.077	0.150	0.054	0.066	0.079	0.100	0.570	0.094	0.067	0.054
0.062	0.048	0.073	0.080	0.081	0.102	0.075	0.081	0.155	0.058	0.072	0.085	0.105	0.620	0.098	0.069	0.057
0.069	0.055	0.082	0.111	0.118	0.118	0.095	0.108	0.181	0.075	0.098	0.125	0.182	0.660	0.125	0.081	0.062
0.078	0.073	0.095	0.151	0.145	0.158	0.118	0.141	0.253	0.161	0.238	0.295	0.381	0.690	0.200	0.112	0.066
0.128	0.138	0.142	0.242	0.238	0.250	0.208	0.225	0.325	0.355	0.395	0.450	0.452	0.715	0.275	0.141	0.071
0.202	0.381	0.225	0.371	0.368	0.380	0.310	0.375	0.458	0.442	0.471	0.495	0.525	0.755	0.361	0.181	0.081
0.402	0.465	0.420	0.391	0.491	0.508	0.550	0.550	0.606	0.481	0.500	0.521	0.565	0.752	0.435	0.268	0.112
0.482	0.511	0.495	0.471	0.542	0.600	0.728	0.740	0.754	0.501	0.519	0.532	0.580	0.771	0.589	0.350	0.123
0.530	0.524	0.522	0.507	0.575	0.618	0.763	0.780	0.797	0.520	0.535	0.550	0.620	0.789	0.673	0.404	0.135
0.540	0.534	0.535	0.517	0.591	0.720	0.790	0.815	0.850	0.530	0.550	0.563	0.680	0.805	0.742	0.495	0.148
0.551	0.545	0.546	0.525	0.608	0.757	0.795	0.822	0.859	0.540	0.561	0.575	0.750	0.820	0.794	0.558	0.161
0.595	0.558	0.557	0.537	0.622	0.785	0.796	0.824	0.863	0.545	0.572	0.588	0.791	0.833	0.832	0.592	0.180
0.578	0.569	0.568	0.549	0.639	0.809	0.796	0.825	0.969	0.548	0.582	0.599	0.820	0.846	0.865	0.616	0.200
0.593	0.580	0.580	0.560	0.651	0.830	0.800	0.830	0.876	0.550	0.591	0.611	0.842	0.000	0.889	0.635	0.220
0.608	0.596	0.593	0.570	0.669	0.847	0.811	0.839	0.883	0.553	0.603	0.625	0.868	0.000	0.910	0.651	0.238

is implied in the following property of $C(\tau)$: it is steep for the small values of τ and rapidly reaches high values. Its steepness is approximately proportional to $\tau \cdot \frac{dr}{d\lambda}$.

The limiting conditions used to obtain the above results actually do not greatly restrict the problem because there are data indicating that conditions I and II really exist in the case of natural formations [2, 3]. Conditions IV and V show certain advantages of the module-structure characteristics in the identification of objects that are similar. This is actually the basic problem underlying each similar algorithm.

The algorithm described in item A and the module-structure characteristics are applied in the following example: the set M consists of 34 reflective characteristics of deciduous and coniferous vegetation as well as grass areas (Table 1). Each reflective characteristics is formed by 40 values of r at each 10 nm in the range of 400-800 nm. These data were taken from paper [4]. The coefficient from condition II is assumed to be 0.02. The application of the algorithm from item A for M shows that the thirty-four classes of M are not identified by means of two-element combinations for λ_i, but that this is possible in 2212 three-element combinations. The same algorithm applied for $C(\tau)$ shows that there exist 29 two-element combinations for the τ values, by means of which the total set M is recognized. As the possible three-element combinations in this case are 9880 and the two-element ones for τ are 190 [$C(\tau)$ is symmetric], the ratios $\frac{2212}{9880}$ and $\frac{29}{190}$ are similar in value Therefore, it can be stated that in this case $C(\tau)$ gives results which are by one order better than $r_j(\lambda_i)$.

It remains to prove the possibilities of the transforming functions C, C_k and K for a set consisting of a considerably larger number of classes.

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Дискриминантный анализ отражательных характеристик естественных природных образований, использующий минимальное число длин волн

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(Резюме)

Рассматривается вопрос о выборе и минимизации необходимого числа длин волн при измерении коэффициента отражения природных образований. Решение этого вопроса позволяет осуществить идентификацию спектральных отражательных характеристик $r_j(\lambda_i)$ данного множества M, состоящего из j классов объектов O_j, $j=1, \ldots N$.